Slides for the course

Statistics and econometrics

Part 12: Causality in a regression framework

European University Institute

Andrea Ichino

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Outline

An extended setup to study the problem

Regression bias for the ATE and for the ATT

Section 1

An extended setup to study the problem

Specification of potential outcomes

Consider the following specification of outcomes, with or without treatment:

$$\begin{aligned} Y_i(1) &= \mu(1) + U_i(1) \\ Y_i(0) &= \mu(0) + U_i(0) \end{aligned} \tag{1}$$

where $E\{U_i(1)\} = E\{U_i(0)\} = 0$. The causal effect of treatment for an individual is

$$\begin{aligned} \Delta_i &= Y_i(1) - Y_i(0) \\ &= [\mu(1) - \mu(0] + [U_i(1) - U_i(0)] \\ &= E\{\Delta_i\} + [U_i(1) - U_i(0)]. \end{aligned} \tag{2}$$

It is the sum of:

- $E\{\Delta_i\} = \mu(1) - \mu(0)$:
  the common gain from treatment equal for every individual i;
- $[U_i(1) - U_i(0)]$:
  the idiosyncratic gain from treatment that differs for each individual i and that may or may not be observed by the individual.

The statistical effects of treatment in this model

1. *The effect of treatment on a random individual (ATE).*

$$\begin{aligned} E\{\Delta_i) &= E\{Y_i(1) - Y_i(0)\} \\ &= E\{Y_i(1)\} - E\{Y_i(0)\} \\ &= \mu(1) - \mu(0) \end{aligned} \tag{3}$$

2. *The effect of treatment on the treated (ATT)*

$$\begin{aligned} E\{\Delta_i \mid D_i = 1) &= E\{Y_i(1) - Y_i(0) \mid D_i = 1\} \\ &= E\{Y_i(1) \mid D_i = 1\} - E\{Y_i(0) \mid D_i = 1\} \\ &= \mu(1) - \mu(0) + E\{U_i(1) - U_i(0) \mid D_i = 1\} \end{aligned} \tag{4}$$

The two effects differ because of the idiosyncratic gain for the treated

$$E\{U_i(1) - U_i(0) \mid D_i = 1\} \tag{5}$$

This is the average gain that those who are treated obtain on top of the average gain for a random person in the population.

A regression with random coefficients

Let D_i indicate treatment: using the equation that relates potential and observed outcomes

$$Y_i = D_i Y_i(1) + (1 - D_i) Y_i(0) \tag{6}$$

we can write

$$\begin{aligned} Y_i &= \mu(0) + [\mu(1) - \mu(0) + U_i(1) - U_i(0)] D_i + U_i(0) \qquad (7) \\ &= \mu(0) + \Delta_i D_i + U_i(0) \end{aligned}$$

where $D_i = 1$ in case of treatment and $D_i = 0$ otherwise.

This is a linear regression with a random coefficient on the RHS variable D_i.

(Figure on board: Differences between treated and control individuals.)

Specification of the selection into treatment

The model is completed by the specification of the rule that determines the participation of individuals into treatment:

$$D_i^* = \alpha + \beta Z_i + V_i \tag{8}$$

where $E\{V_i\} = 0$ and

$$D_i = \begin{cases} 1 & \text{if } D_i^* \geq 0 \\ 0 & \text{if } D_i^* < 0 \end{cases} \tag{9}$$

D_i^* is the (unobservable) criterion followed by the appropriate decision maker concerning the participation into treatment of individual i. The decision maker could be nature, the researcher or the individual.

Z_i is the set of variables that determine the value of the criterion and therefore the participation status. No randomness of coefficients is assumed here.

Z_i could be a binary variable.

The structural model in compact form

$$Y_i = \mu(0) + \Delta_i D_i + U_i(0) \tag{10}$$

$$D_i^* = \alpha + \beta Z_i + V_i \tag{11}$$

$$D_i = \left\{ \begin{array}{ll} 1 & \text{if } D_i^* \geq 0 \\ 0 & \text{if } D_i^* < 0 \end{array} \right\} \tag{12}$$

$$\begin{aligned} \Delta_i &= \mu(1) - \mu(0) + U_i(1) - U_i(0) \\ &= E\{\Delta_i\} + U_i(1) - U_i(0) \end{aligned} \tag{13}$$

$$E\{U_i(1)\} = E\{U_i(0)\} = E\{V_i\} = 0 \tag{14}$$

Correlation between U_i and V_i is possible.

Section 2

Regression bias for the ATE and for the ATT

Bias for the effect of treatment on a random person

Using 13 we can rewrite equation 10 as:

$$\begin{aligned} Y_i &= \mu(0) + E\{\Delta_i\}D_i + U_i(0) + D_i[U_i(1) - U_i(0)] \\ &= \mu(0) + E\{\Delta_i\}D_i + \epsilon_i \end{aligned} \tag{15}$$

that tells us what we get from the regression of Y_i on D_i.

Problem:

$$E\{\epsilon_i D_i\} = E\{U_i(1) \mid D_i = 1\}Pr\{D_i = 1\} \neq 0 \tag{16}$$

Therefore the estimated coefficient of Y_i on D_i is a biased estimate of $E\{\Delta_i\}$

$$E\{Y_i \mid D_i = 1\} - E\{Y_i \mid D_i = 0\} = E\{\Delta_i\} + \tag{17}$$

$$E\{U_i(1) - U_i(0) \mid D_i = 1\} + E\{U_i(0) \mid D_i = 1\} - E\{U_i(0) \mid D_i = 0\}$$

The second line is the bias for the ATE

Analysis of the bias for the ATE

Readjusting the second line of 17, the bias can be written as:

$$E\{Y_i \mid D_i = 1\} - E\{Y_i \mid D_i = 0\} = E\{\Delta_i\} + \tag{18}$$

$$E\{U_i(1) \mid D_i = 1\} - E\{U_i(0) \mid D_i = 0\}$$

This bias is equal to the difference between two componenents:

- $E\{U_i(1) \mid D_i = 1\}$
  the unobservable outcome of the treated in case of treatment;
- $E\{U_i(0) \mid D_i = 0\}$
  the unobservable outcome of the controls in the case of no treatment.

In general, there is no reason to expect this difference to be equal to zero.

Consider a controlled experiment in which participation into treatment is random because

- assignment to the treatment or control groups is random and
- there is full compliance with the assignment.

Is the bias for the ATE interesting for policy?

Under these assumptions it follows that:

$$\begin{aligned} E\{U_i(1)\} &= E\{U_i(1) \mid D_i = 1\} = 0 \\ E\{U_i(0)\} &= E\{U_i(0) \mid D_i = 0\} = 0 \end{aligned} \tag{19}$$

Hence, under perfect randomization, the treatment and the control groups are statistically identical to the entire population and therefore

$$\begin{aligned} E\{\Delta_i\} &= E\{Y_i(1)\} - E\{Y_i(0)\} \\ &= E\{Y_i(1) \mid D_i = 1\} - E\{Y_i(0) \mid D_i = 0\} \\ &= \mu(1) - \mu(0) \end{aligned} \tag{20}$$

But, is the effect of treatment on a random person interesting in economic examples?

Bias for the effect of treatment on a treated person

Adding and subtracting $D_i E\{U_i(1) - U_i(0) \mid D_i = 1\}$ in 15 and remembering from 4 that $E\{\Delta_i \mid D_i = 1\} = E\{\Delta_i\} + E\{U_i(1) - U_i(0) \mid D_i = 1\}$, we can rewrite 15 as:

$$\begin{aligned} Y_i &= \mu(0) + E\{\Delta_i \mid D_i = 1\}D_i + \\ &\quad U_i(0) + D_i[U_i(1) - U_i(0) - E\{U_i(1) - U_i(0) \mid D_i = 1\}] \\ &= \mu(0) + E\{\Delta_i \mid D_i = 1\}D_i + \eta_i \end{aligned} \tag{21}$$

Using 21 we can define the OLS bias in the estimation of $E\{\Delta_i \mid D_i = 1\}$.

$E\{\Delta_i \mid D_i = 1\}$ is the ATT which is equal to the common effect plus *the average idiosyncratic gain*.

The error term is again correlated with the treatment indicator D_i:

$$\begin{aligned} E\{\eta_i D_i\} &= E\{D_i U_i(0) + D_i[U_i(1) - U_i(0) - E\{U_i(1) - U_i(0) \mid D_i = 1\}]\} \\ &= E\{D_i U_i(0)\} \neq 0. \end{aligned} \tag{22}$$

Analysis of the bias for the ATT

Therefore, the estimated coefficient of Y_i on D_i is biased also with respect to $E\{\Delta_i \mid D_i = 1\}$:

$$E\{Y_i \mid D_i = 1\} - E\{Y_i \mid D_i = 0\} = E\{\Delta_i \mid D_i = 1\} + \tag{23}$$

$$E\{U_i(0) \mid D_i = 1\} - E\{U_i(0) \mid D_i = 0\}$$

The second line in 23 is the bias for the ATT

$$E\{U_i(0) \mid D_i = 1\} - E\{U_i(0) \mid D_i = 0\}$$

is called *mean selection bias* and “tells us how the outcome in the base state differs between program participants and non-participants.

Absent any general equilibrium effects of the program on non participants, such differences cannot be attributed to the program.” (Heckman, 1997)

This bias is zero only when participants and non-participants are identical in the base state i.e. when $E\{U_i(0)D_i\} = 0$.

Would randomization help in the estimation of the ATT?